Generalized modes in Bayesian inverse problems

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Our work is concerned with non-parametric modes and MAP estimates for priors that do not admit continuous densities, for which previous approaches based on small ball probabilities fail. We propose a novel definition of generalized modes based on the concept of qualifying sequences, which reduce to the classical mode in certain situations that include Gaussian priors but also exist for a more general class of priors. The latter includes the case of priors that impose strict bounds on the admissible parameters and in particular of uniform priors. For uniform priors defined by random series with uniformly distributed coefficients, we show that the generalized modes – but not the classical modes – as well as the corresponding generalized MAP estimates can be characterized as minimizers of a suitable functional. This is then used to show consistency of nonlinear Bayesian inverse problems with uniform priors and Gaussian noise.

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