New convergence rates for variational Lavrentiev regularization of nonlinear monotone ill-posed problems

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We consider nonlinear ill-posed equations $Fu = f$ in Hilbert spaces $\mathcal{H}$, where the operator $F : \mathcal{H} \to \mathcal{H}$ is monotone on a closed convex subset $\mathcal{M} \subset \mathcal{H}$. For given data $f^\delta \in \mathcal{H}, \|f^\delta - f\| \le \delta$, a standard approach is Lavrentiev regularization $Fv_\alpha^\delta + \alpha v_\alpha^\delta = f^\delta$, with $\alpha > 0$ small and appropriately chosen, and $v_\alpha^\delta \in \mathcal{M}$ has to be determined. However, in practical applications like parameter estimation problems, solvability of the regularized equation on the monotonicity set $\mathcal{M}$ cannot be guaranteed, in general. We thus consider a regularized variational inequality instead, i.e., determine $u_\alpha^\delta \in \mathcal{M}$ satisfying

$$\langle Fu_\alpha^\delta + \alpha u_\alpha^\delta - f^\delta, w - u_\alpha^\delta \rangle \ge 0 \quad \text{for each } w \in \mathcal{M}.$$

In this talk we present new estimates of the error $u_\alpha^\delta - u_*$ for suitable choices of $\alpha = \alpha(\delta)$, if the solution $u_* \in \mathcal{M}$ of $Fu = f$ is source-representable. This is joint work with B. Hofmann (TU Chemnitz).

References:

[1] Plato, R., and Hofmann, B: Convergence rates of a penalized variational inequality method for nonlinear monotone ill-posed equations in Hilbert spaces, *arXiv:1806.00743*, June 2018.

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