On the connection between sparsity and tolerances in parameter and data space

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Tikhonov regularization for the solution of ill-posed inverse problems is well established. In convergence theory, one is interested in studying the convergence of Tikhonov minimizers when the noise level tends to zero. Another interesting question is under which assumptions convergence rates can be derived, or even improved.

In applications, sparsity regularization has attracted much attention, and convergence rates for sparsity-promoting Tikhonov functionals with fixed norms have already been proved. It has also been shown how support vector regression can be used for the solution of ill-posed inverse problems in reproducing kernel Hilbert spaces. For this purpose, a Tikhonov functional with a tolerance function is used. The discrepancy between the operator evaluation and the given data is measured using the tolerance function.

Some applications in the fields of production engineering and new material development require that one deals with deviations in parameter and data space. Since some first results already exist in the case of deviations in the data space, we focus on reconstructing parameters that lie within a confidence interval. The tolerance function is used so that deviations within a prescribed tolerance area can be neglected.

We study the behavior of minimizers for an altered Tikhonov functional with a tolerance function introduced in the regularization term. We examine the particularly interesting case when both the noise level and the tolerance tend to zero. In addition, we discuss the connection of our setting to sparsity regularization.

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