Inverse source problems for partial differential equations with data after the incidents

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Let an evolution equation be written in a general form

$$\partial_t^k u(x,t) = Au(x,t) + \lambda(t)f(x),$$

$k = 1$ or $= 2$ in $Q := \Omega \times (0,T)$ with the zero boundary and initial conditions. We consider the inverse source problem:
Given $f(x) \not\equiv 0$ and a monitoring point $x_0 \in \Omega$ and $0 \le T_1 < T_2$, determine $\lambda(t)$, $0 < t < T$ by $u(x_0,t)$, $T_1 < t < T_2$.

In the case of $T_1 = 0$ and $T_2 = T$, the inverse problem is classical and the researches have been satisfactorily done. However, in practice, it is often that $\lambda(t) = 0$ for $t > T$ and $0 < t < T < T_1 < T_2$. This means that data are available only after the finish of the incident such as explosion. One of our results is that in the case where Ω is one-dimensional, we cannot have the uniqueness in the inverse problem but in the case where the dimensions are greater than or equal to 2, the uniqueness holds. Moreover we give various formulations of inverse source problems and show several results on the uniqueness and the stability.

The talk is based on joint works with Professor J. Cheng (Fudan Univ.) and Professor S. Lu (Fudan Univ.).

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