A new semi-algebraic two-grid method for Oseen problems

Pierre-Loïc Bacq[1] Yvan Notay[2]

We consider the numerical solution of discrete Oseen problems. These problems naturally arise when linearizing Navier-Stokes problems with the Picard method and are challenging to solve[1]: as far as we know, no linear-time method can robustly solve them in a number of iterations bounded independently of the mesh size and of the Reynolds number, especially for variable convective flows. In particular, algebraic multigrid (AMG) methods often struggle with such problems because of the small or null pressure block[2].

The approach we propose is based on a simple algebraic transformation of the corresponding linear system and has already given good results for Stokes problems[3]. In this setting, our contribution is twofold. First, we present a norm-based algebraic convergence theory for the two-grid method applied to the transformed system. For constant coefficient problems discretized by finite differences, we show that the convergence is indeed independent of both the mesh size, the Reynolds number and the orientation of the convection flow. To obtain this result, unsmoothed aggregation is used and aggregates of both velocity and pressure unknowns should be aligned in the direction of the convective flow.

Our second contribution is a semi-algebraic multigrid method which is applied to the transformed system. This method is intended for problems with variable convective flow. Information is used from the discretization to build an auxilary matrix, from which the aggregation of the pressure unknowns is algebraically determined. The aggregation of the velocity unknowns is algebraically determined from the velocity block of the system matrix. Numerical results show that the two-grid method converges independently of the mesh size and of the Reynolds number for constant convective flows, and almost independently of the Reynolds number for variable flows.

References:

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[1]Université Libre de Bruxelles
pierre-loic.bacq@ulb.be

[2]Université Libre de Bruxelles
yvan.notay@ulb.be