Randomised preconditioning for time-parallel data assimilation

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Data assimilation combines observations of a dynamical system with a prior estimate of its state to obtain the best possible state estimate. The state can be described by as many as 10^9–10^{10} variables and 10^8 observations are assimilated in applications like numerical weather prediction, where data assimilation provides initial conditions for the numerical model.

We consider the incremental weak-constraint four-dimensional variational data assimilation method, where series of large sparse linear systems of equations are solved. The 3x3 block saddle point formulation of the systems is suitable for time-parallel computations. These large sparse systems are solved using Krylov subspace solvers and efficient preconditioning that preserves the potential for time-parallelism is essential.

In this work, we focus on the block diagonal preconditioner that includes approximation to the inverse of the Schur complement. This approximation usually disregards observation information, and we propose a new way to incorporate it. This is achieved by computing a randomised eigenvalue decomposition of the Schur complement and using it to construct a spectral limited memory preconditioner. Such approximation preserves time-parallelism when generating and applying the preconditioner. We also analyse how the extreme eigenvalues of the preconditioned coefficient matrix change when new observations are introduced. An idealised numerical example illustrates the theory and shows that the new preconditioner is effective.

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