Block Toeplitz matrices in domain decomposition

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Quantifying the convergence and proving scalability of Schwarz domain decomposition methods in the many subdomains case for problems where the underlying partial differential operator is non-self adjoint, can often be complicated and require very restrictive assumptions. In this work we revisit the iterative version of these methods for simple decompositions in many subdomains. By taking advantage of the the block Toeplitz structure of iteration matrices appearing naturally in the algorithm we can exploit the notion of limiting spectrum to characterise the full spectrum of these iteration matrices and ultimately prove the convergence and scalability. The notion of limiting spectrum can also be used to derive new optimised domain decomposition methods and obtain for the first time, closed forms of interface transmission conditions in the many subdomain case. We apply these ideas to the Helmholtz, two-dimensional Maxwell and complex- diffusion equations (which can be seen as the magneto telluric approximation of Maxwell's equations). We will illustrate our findings on some well-chosen test cases on uniform and METIS decompositions into subdomains.

References:

[1] https://doi.org/10.1553/etna_vol55s112

[2] https://arxiv.org/abs/2204.09754

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