Level 2 Circulant Preconditioners for the Navier-Stokes Equations

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We evaluate several preconditioning strategies for the stream function form of the Navier-Stokes equations discretised by C^1 Bogner-Fox-Schmit finite elements. In this finite element, each mesh node has four degrees of freedom (DOF). We start by ordering the interior DOFs then the boundary DOFs. Based on this ordering, The Jacobian matrix A has a 2×2 block structure. Since the stream function equation is a fourth order pde, the condition number of the matrix A behaves as $\kappa = O(h^{-4})$ where h is the meshsize [1]. Unfortunately, convergence speed of any Krylov subspace iterative methods is effected by this rapid growth in the condition number.

The upper left submatrix of the Jacobian matrix is block-tridiagonal and block-Toeplitz matrix. With each block being block-tridaiagonal and block-Toeplitz matrix. Based on this matrix structure, we use best circulant approximation [2] to develop Level 1 block-circulant and Level 2 block-circulant preconditioners. One of our motivations is to use the fast inversion of block-circulant systems via the two-dimensional Fast Fourier Transform (FFT2). Numerical experiments show that the preconditioned matrices have clustering of eigenvalues. The efficiency of these preconditioners are confirmed via the preconditioned residual plots.

References:

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[2] Pestana J, Muddle R, Heil M, Tisseur F, Mihajlovic M. Efficient Block Preconditioning for a C^1 Finite Element Discretization of the Dirichlet Biharmonic Problem. SIAM Journal on Scientific Computing. 2016;38(1):A325-45.

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