Preconditioning Optimal Control of Incompressible Viscous Fluid Flow

Santolo Leveque[1] John W. Pearson[2]

Optimal control problems with PDEs as constraints arise very often in scientific and industrial problems. Due to the difficulties arising in their numerical solution, researchers have put a great effort into devising robust solvers for this class of problems. An example of a highly challenging problem attracting significant attention is the (distributed) control of incompressible viscous fluid flow problems. In this case, the physics may be described, for very viscous flow, by the (linear) incompressible Stokes equations, or, in case the convection of the fluid plays a non-negligible role in the physics, by the (non-linear) incompressible Navier–Stokes equations. In particular, as the PDEs given in the constraints are non-linear, in order to obtain a solution of Navier–Stokes control problems one has to iteratively solve linearizations of the problems until a prescribed tolerance on the non-linear residual is achieved.

In this talk, we present novel, efficient, and robust preconditioned iterative methods for the solution of the time-dependent incompressible Stokes and Navier–Stokes control problems, with backward Euler discretization in time. The proposed preconditioner is based on a saddle-point type of approximation. We employ an inner iteration for the $(1,1)$-block accelerated by preconditioned iterative methods for heat and convection–diffusion control problems when employing backward Euler in time. In addition, in order to derive an approximation for the Schur complement we employ a potent and flexible commutator argument applied to an appropriate block matrix. The flexibility of the commutator argument, which is a generalization of the technique derived in [1], allows one to alternatively apply a Crank–Nicolson discretization in time, as well as to solve the stationary regimes for the problems considered. We show the effectiveness and robustness of our approach through a range of numerical experiments.

This talk is based on the work in [2] and [3].

References:

[1] D. Kay, D. Loghin and A. J. Wathen, *A Preconditioner for the Steady-State Navier–Stokes Equations*, SIAM Journal on Scientific Computing **24**, 237–256, 2002.

[2] S. Leveque and J. W. Pearson, *Parameter-Robust Preconditioning for Unsteady Stokes Control Problems*, Proceedings in Applied Mathematics and Mechanics **21**, e202100131, 2021.

[3] S. Leveque and J. W. Pearson, *Parameter-Robust Preconditioning for Oseen Iteration Applied to Stationary and Instationary Navier–Stokes Control*, to appear in SIAM Journal on Scientific Computing, arXiv preprint arXiv:2108.00282.

[1]University of Edinburgh, School of Mathematics, Edinburgh
s1888324@ed.ac.uk

[2]University of Edinburgh, School of Mathematics, Edinburgh
j.pearson@ed.ac.uk