Robust preconditioning for a mixed formulation of phase-field fracture problems

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We consider fracture propagation in nearly incompressible and (fully) incompressible materials using a phase-field formulation. We use a mixed form of the elasticity equation to overcome volume locking effects and develop a robust, nonlinear and linear solver scheme and preconditioner for the resulting system. The coupled variational inequality system, which is solved monolithically, consists of three unknowns: displacements, pressure, and phase-field. Nonlinearities due to coupling, constitutive laws, and crack irreversibility are solved using a combined Newton algorithm for the nonlinearities in the partial differential equation and employing a primal-dual active set strategy for the crack irreverrsibility constraint. The linear system in each Newton step is solved iteratively with a flexible generalized minimal residual method (GMRES). The key contribution is the development of a problem-specific preconditioner that leverages the saddle-point structure of the displacement and pressure variable. Four numerical examples in pure solids and pressure-driven fractures are conducted on uniformly and locally refined meshes to investigate the robustness of the solver concerning the Poisson ratio as well as the discretization and regularization parameters.

References:

[1] https://arxiv.org/abs/2202.04191

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