Advances in Block Preconditioning for Time-Dependent PDE-Constrained Optimization

John Pearson[1]

Optimization problems subject to PDE constraints form a class of problems which may be applied to a wide range of scientific processes, including fluid flow control, medical imaging, biological and chemical processes, and electromagnetic inverse problems, to name a few. These problems involve minimizing some function arising from a particular physical objective, while at the same time obeying a system of PDEs which describe the process. It is crucial to be able to obtain accurate numerical solutions to such problems within a reasonable CPU time, in particular for time-dependent problems, for which the "all-at-once" solution can lead to extremely large linear systems once a suitable spatial discretization and time-stepping scheme have been applied.

In this talk we tackle this important challenge by applying fast and robust preconditioned Krylov subspace methods. We survey research into efficient numerical methods and block preconditioners for the accurate solution of a number of practical problems, with particular contributions including: (i) block saddle-point preconditioners for linear systems arising from large-scale time-dependent problems; (ii) a spectral-in-time Newton–Krylov method for solving nonlinear, time-dependent optimization problems to high accuracy, by applying column operations to the Jacobian matrix at each Newton iteration along with a Kronecker product-based preconditioner for the resulting Schur complement; (iii) iterative solution of optimization problems constrained by certain space–time fractional differential equations using multilevel circulant preconditioners, that lead to very reasonable storage requirements and computational operation costs.

[1]University of Edinburgh
j.pearson@ed.ac.uk