A double Schur complement approach for preconditioning certain regularized saddle-point systems

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The recently proposed Sequential Homotopy Method (Potschka, Bock 2021) can be applied to solve challenging nonlinear optimal control problems with partial differential equations (PDEs). Using a local semi-smooth Newton solver for the nonlinear homotopy subproblems gives rise to a sequence of active-set dependent, regularized, linear saddle-point systems. Their structure can be exploited to construct efficient preconditioners for Krylov-subspace methods. For a certain class of optimal control problems with PDE constraints, in which the control enters the Lagrangian only linearly, we propose and analyze an efficient, parallelizable, symmetric positive definite preconditioner based on a double Schur complement approach. We conclude with numerical results for a badly conditioned and highly nonlinear benchmark optimization problem with elliptic partial differential equations and control bounds. The resulting method is faster than using direct linear algebra for the 2D benchmark and allows for the parallel solution of large 3D problems.

References:

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