

FROSch – the Fast and Robust Overlapping Schwarz Preconditioner framework – in Chemo-Mechanics

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In chemo-mechanics the balance relations of solid mechanics are coupled to (reaction-)diffusion equations. Applications include biomechanics, e.g. cell growth, safety assessment of engineering structures subject to hydrogen diffusion, cross-linking kinetics, or swelling of hydrogels. An incremental variational formulation of this multi-field problem [Boeger, 2017] is adopted here, which exploits the underlying minimization structure. The corresponding weak form is implemented using the finite element library deal.II [Arndt, 2021]. The monolithic linear system resulting from Newton linearization is solved using the Fast and Robust Overlapping Schwarz (FROSch) framework of the Trilinos software library. FROSch implements parallel overlapping Schwarz domain decomposition preconditioners with an energy minimizing coarse space of GDSW type [Heinlein, 2022]. The parallel scalability of the preconditioner – operating in algebraic mode – is assessed for some characteristic examples employing both conforming and non-conforming finite element discretizations.

References:

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