Element-based preconditioners for mixed finite element problems

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In this talk we introduce a new and generic approximation to Schur complements arising from inf-sup stable mixed finite element discretizations of self-adjoint multi-physics problems. In recent years 'natural-norm' preconditioning, where the preconditioner is inferred from the function space on which the underlying problem is posed, has proved to be a successful approach. However, this requires detailed knowledge of the problem being solved.

Our new approach builds a sparse approximation to a natural-norm preconditioner. It does this by exploiting the discretization mesh by forming local, or element, Schur complements and projecting them back to the global degrees of freedom. The resulting Schur complement approximation is sparse, has low construction cost (with the same order of operations as a general finite element matrix), and can be solved using off-the-shelf techniques, such as multigrid.

Using the LBB condition, we show that the preconditioned system has a favourable eigenvalue distribution. We present several numerical results to demonstrate the viability of this approach on a range of applications. Interestingly, numerical results show that the method gives an effective approximation to the non-symmetric Schur complement from the steady-state Navier-Stokes equations.

References:

[1] https://epubs.siam.org/doi/abs/10.1137/20M1336461

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